Scalability of Airborne Wind Energy Systems

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ABSTRACT

The scalability of airborne wind energy systems with crosswind wing motion is analyzed. The main dimensions of AWES are found to be linearly scalable in the first approximation. The power grows as a square of the linear dimensions of AWES. Scaling up may provide additional benefits, such as lower acceleration of a wing and a tether in a closed loop, decreased tether drag when the tether Reynolds number becomes critical, and well known statistical increase of wind energy density with altitude. Further, the paper shows that at a reasonably high ratio of the wing area to the distance between the wing and its ground attachment, the tether drag becomes significantly lower than the wing drag. The paper introduces an assumption of an “efficient AWES”, which makes analysis easier without limiting practical applicability of the results.
Nomenclature

\( \phi \) angle between tension force and horizon
\( \sigma \) tensile strength of the tether material
\( \eta \) \( D_I/D_k \) ratio
\( \rho \) air density at the altitude of the wing
\( \tau \) torque in the drum
\( A \) wing surface area
\( A_{t,I} \) effective drag area of tether
\( A_{t,s} \) cross-sectional areas of tether
\( b \) wingspan
\( C_L \) wing lift coefficient
\( C_D \) wing drag coefficient (wing alone—not including tether drag)
\( C_t \) tether form drag coefficient
\( D_k \) wing drag
\( D_t \) tether drag
\( g \) \( g \)
\( L \) lift force
\( l_0 \) distance between the wing and its ground attachment (wing distance)
\( d \) width of tether
\( P \) power
\( q \) radius of circular trajectory of the wing
\( R \) wing aspect ratio
\( r \) radius of a drum
\( s \) safety margin
\( T \) tension force
\( V \) wind velocity
\( V_{\phi} \) \( V \cos \phi \)
\( W \) relative velocity through air
1. Introduction

Recently, the airborne wind energy field has experienced significant progress in research, development and commercialization. The overview of the field can be found in [1] and [2]. Airborne wind energy systems (AWES) have some obvious advantages compared with conventional wind turbines ([3]) and other renewable energy sources ([4]). Some AWES designs were theoretically proven to be 10 times cheaper than conventional wind energy systems with the same average power ([5], [6]). Nevertheless, scalability of AWES has never been rigorously addressed in the published papers, causing unnecessary R&D expense by practitioners and leaving a gap in the public perception of airborne wind energy. This paper discusses relationships between linear parameters of AWES and rigorously proves linear scalability of properly designed AWES.

2. Scalability of Tethered Wing

2.1. Derivation of the relation between wingspan and wing distance

We will consider an efficient airborne wind energy system with a single tethered wing operating at nominal wind speed \( V \) at maximum Loyd’s efficiency ([7]). Here, “efficient” means that the tether safety margin is not too large (\( s < 4 \)). Any practical AWES is expected to be efficient in this sense. Some technical approaches to developing such AWES are discussed in [8].

Fig. 1. AWES with circular motion of the wing. The ground attachment may be a drum coupled to an electrical generator.

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In this system, the formulas for the wing lift and drag are usual:

\[ L = \frac{1}{2} C_L \rho A W^2 \]  
(1)

\[ D_k = \frac{1}{2} C_D \rho A W^2 \]  
(2)

The formula for the tether drag was derived in [9] as equation 15. In our notation, it is expressed as

\[ D_t = \frac{1}{8} C_t \rho A_{t,l} W^2 \]  
(3)

Effective drag area of the tether \( A_{t,l} \) is the area tether projection to the plane, perpendicular to relative air velocity. It should be noted, that besides tension, tether experiences drag, inertia and weight. Forms of the tether projection in different planes can be easily computed using variable weight catenary model. It is easy to show, that tether curvature due to weight and inertia are relatively small, if the system is designed for efficiency and is operated at the nominal wind. Thus the effective drag area can be estimated as

\[ A_{t,l} = l_D d \]  
(4)

The cross-sectional area of the tether is given by the following expression:

\[ A_{t,s} = \frac{sT}{\sigma} \]  
(5)

where maximum tension \( T \) is approximately equal to lift \( L \). The perpendicular cross-sectional area is expressed as a function of the tether width \( d \) as follows:

\[ A_{t,s} = f d^2 \]  
(6)

where \( f \) is a coefficient that depends on the sectional form of the cable. For example, \( f = \pi/4 \) for round tethers. Formula (6) allows us to compute \( A_{t,l} \) and substitute it into (3), yielding:

\[ D_t = \frac{1}{8} C_t \rho l_D \sqrt{\frac{C_L \rho A_s}{2\sigma}} W^3 \]  
(7)

Let us define:

\[ \eta = \frac{D_t}{D_k} \]  
(8)

The system developer would want to keep \( \eta \) low to minimize total drag \( D_t + D_k \). Using the definition of aspect ratio \( R \):

\[ R = \frac{b^2}{A} \]  
(9)

we obtain
\[ \eta = \frac{C_L}{4C_D} \frac{l_D}{b} \sqrt{\frac{C_L \rho sR}{2\pi \sigma}} W \] (10)

Based on [9], the glide ratio is expressed as

\[ G = \frac{C_L}{C_D(1+\eta)} \] (11)

From Loyd, [7]:

\[ W = \frac{2}{3} \frac{l_D}{D_D} V \varphi = \frac{2}{3} \frac{C_L}{C_D(1+\eta)} V \varphi \] (12)

Substituting \( W \) from (12) and \( f = \pi/4 \) into (10), we obtain:

\[ \eta = \frac{C_L}{C_D(1+\eta)} \frac{l_D}{b} \frac{C_L}{3C_D} \sqrt{\frac{C_L \rho sR}{2\pi \sigma}} V \cos \varphi \] (13)

From which we can express the ratio of the wing distance to the wingspan:

\[ \frac{l_D}{b} = \frac{3\eta}{\sqrt{2V \cos \varphi}} \sqrt{\frac{2\pi \sigma}{C_L \rho sR}} \frac{C_D}{C_D(1+\eta)} C_L \] (14)

At practical glide ratio values, the tether length is only slightly larger than the wing distance. Formula (14) is not dependent on the power removal method. \( C_L \) and \( C_D \) are constant for the applicable range of the wing Reynolds numbers. For a rough tether and applicable tether Reynolds numbers, \( C_L \) is within the range 0.4 – 1.2. Air density \( \rho \) decreases slowly with altitude, and even this variation is compensated by statistically increasing wind velocities. Thus, the wing distance to the wingspan ratio is approximately constant for any selected \( \eta \).

Thickness \( d \) of the round tether is expressed from (1), (5) and (6) as

\[ d = \frac{2}{3} \sqrt{\frac{2C_L \rho s}{\pi \sigma R}} \frac{C_L}{C_D(1+\eta)} b V \cos \varphi \] (15)

Thus, the tether thickness is proportional to the wingspan. The power is proportional to the square of the wingspan \( b \):

\[ P = \frac{2}{27} \frac{C_L}{C_D(1+\eta)}^2 \frac{\rho}{R} b^2 V^3 \cos^2 \varphi \] (16)

### 2.2. Discussion of the real world values

Table 1 shows typical values of the discussed parameters. It is easy to see from (13) that an increase in the \( l_D/b \) causes a decrease in \( \eta \) and improves glide ratio. But there are some limitations on the wingspan. Without loss of generality, let us assume that the wing’s trajectory is a circle or a helix with a small pitch-to-radius ratio, which can be approximated by a circle in a plane, perpendicular to the vector...
ground attachment - wing. [10] states that radius $q$ of this circle can be as little as 2.5 wing spans, based on experiments with leading edge inflatable kites. In fact, the radius is also limited by the maximum acceleration that the wing (including its control unit) can survive:

$$q \geq \frac{W^2}{a_{\text{max}}}$$

Table 1. Examples of AWES with a kite and glider wings.

<table>
<thead>
<tr>
<th></th>
<th>Kite Wing</th>
<th>Glider Wing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>0.25</td>
<td>1.1</td>
</tr>
<tr>
<td>$V$, m/s</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>$\varphi$, degrees</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>$\rho$, kg/m$^3$</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>$C_L$</td>
<td>1.2</td>
<td>1.5</td>
</tr>
<tr>
<td>$C_D$</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma$, Pa</td>
<td>3.0E+09</td>
<td>3.0E+09</td>
</tr>
<tr>
<td>$C_t$</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>$s$</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$R$</td>
<td>6.0</td>
<td>12.0</td>
</tr>
<tr>
<td>$V_\varphi$, m/s</td>
<td>13.0</td>
<td>13.0</td>
</tr>
<tr>
<td>$C_L/C_D$</td>
<td>8.0</td>
<td>30.0</td>
</tr>
<tr>
<td>$C_L/C_{\text{drag total}}$</td>
<td>6.4</td>
<td>14.3</td>
</tr>
<tr>
<td>$l_D/b$ (wing distance to wingspan)</td>
<td>37.2</td>
<td>15.5</td>
</tr>
<tr>
<td>$l_D$, m (for power computations)</td>
<td>1,000.0</td>
<td>500.0</td>
</tr>
<tr>
<td>$b$, m</td>
<td>26.9</td>
<td>32.3</td>
</tr>
<tr>
<td>$A$, m$^2$</td>
<td>120.16</td>
<td>86.97</td>
</tr>
<tr>
<td>$P$, kW</td>
<td>1150.8</td>
<td>5187.9</td>
</tr>
</tbody>
</table>
Taking, for example, a production unit with \( W = 100 \text{ m/s} \) and \( a_{\text{max}} = 10g \), we obtain \( q \geq 100 \text{ m} \). That means that this limitation will not be a factor in production units above a certain small size.

Nevertheless, too tight a circle represents other problems: decreased effective wing surface because of the angle required to impart centrifugal force to the wing and large variation in the relative air speed at the wingtips. The latter problem can be overcome by employing an asymmetrical wing, as suggested in [11]. Among other system limitations: the altitude of the center of the circle \( h \) should be at least twice the circle radius, otherwise variation in the wind speed and direction at the top and the bottom of the circle will be too large. (Note that the altitude \( h \) also may be variable.) Finally, in order not to lose too much power to \( \cos(\phi) \) in (15), we should require \( l_0/h > 2 \). Summarizing the requirements in this subsection, we arrive at a limitation

\[
\frac{l_0}{b} > 10 \tag{18}
\]

Note that \( l_0/b \) near the low boundary still involves painful compromises (which are not reflected in Table 1). For example, Makani Power has selected \( q/b \) ratio in the range 4-5 ([12]), although this choice might be less relevant here because onboard generators in Makani design impose their own limitations. Table 1 also demonstrates that for a glider wing with high \( L/D \) ratio, the tether drag may exceed the wing drag. Makani Power probably knew this fact in 2008, as indicated by its proposal of a streamlined tether in [13].

Table 1 uses a “typical” \( C_t \) for a cylinder in a laminar flow that holds for a wide range of Reynolds numbers. Some teams, experimenting with smaller and/or slower kites might encounter significantly higher \( C_t \), for which the results of this paper do not apply. On the other hand, the theory predicts that tether drag coefficient will drop up to 2-3 times when the tether Reynolds number becomes critical.

Reynolds number will become critical with increase in the tether thickness and air speed. Thus, efficiency of AWES is expected to increase with increase in its linear dimensions due to at least three effects, not captured by the linear first approximation: increase in the statistical wind energy density at higher altitudes, decrease in turn acceleration and likely drop in the tether drag coefficient.

### 3. Scalability of Drivetrain

Today, the most common method of transferring harvested wind power to the ground generator is by letting the tether unreel from a drum, the drum converting the linear motion of the tether to the rotational motion. The drum is coupled to the rotor of the generator via a gearbox. This system is not
scalable, as was mentioned in [6] and explained in details below. From the formula for angular speed and Loyd’s optimal tether reel-out speed:

\[ \omega = \frac{V \cos \varphi}{6\pi r} \]  \hspace{1cm} (19)

Torque acting on the drum is

\[ \tau = \frac{P}{\omega} \]  \hspace{1cm} (20)

The drum radius \( r \) is proportional to the thickness of the tether \( d \), which is proportional to the wingspan \( b \). Thus, the angular speed of the drum decreases with an increase in the power, while the torque, acting on the drum and the gear, increases faster than the power.

\[ \omega \sim \frac{1}{b} \sim \frac{1}{\sqrt{P}} \]  \hspace{1cm} (21)

\[ \tau \sim b^3 \sim P^{\frac{3}{2}} \]  \hspace{1cm} (22)

Fortunately, there is a simple solution for the problem (first published in [14]): decouple the drum from the rotor, use a flat perforated belt or chain, connected to the tether, and let the belt or chain engage a gear or a sprocket, coupled to the rotor, as shown in Fig. 2:
As long as the thickness of the belt is kept constant when the nominal power increases, the angular speed of the gear remains constant, and the torque grows linearly with the power. An even better solution is provided by using separate motion transfer, as proposed in [5] and [6], where the angular speed is not only constant but sufficiently high to dispense with a gearbox.

4. Conclusions

The most important practical conclusion is linear scalability of the main dimensional parameters of the airborne wind energy systems with the ground generator: the distance of the wing to the ground
station, the wingspan and the tether thickness (provided the same aspect ratio of the wing, angle $\varphi$, nominal wind speed and air density). This allows a practicing engineer to perfect the control of the wing on a small system, and then immediately proceed to a production model with a many times larger wing area and power output.

Another practically important conclusion is that it is easy to achieve a relatively small contribution of the tether drag to the total drag by increasing the wing area.

Formula (18) establishes the low boundary for the ratio of the wing distance to the wingspan. Formula (14) is derived to compute the ratio of the wing distance to wingspan.

As a contribution to the theory, formula (11) eliminates dependence of the total drag ratio on the tether drag.

5. Disclosure statement

The author has pending patent applications related to the article content.

6. References


